Non-linear mean, or non-linear variance, or both? A case study in Australian electricity markets

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Abstract

With the introduction of competitive markets in the electricity industry, there is a growing demand for research in spot price modeling and forecasting. The objective of this paper is to test the hypothesis that both the conditional mean and conditional variance of electricity spot price changes are asymmetric functions of past information. For this purpose, a bilinear (BL) model with BL-GARCH errors is introduced and estimated for the Australian national electricity market (NEM). The empirical evidence suggests that the conditional mean responds asymmetrically to past information in New South Wales and Queensland electricity market. For Victoria, the conditional mean dynamics appear to be largely linear. Further, in agreement with other studies, our study indicates fairly strong evidence of asymmetries in the conditional variance for all electricity spot markets.

Keywords: Asymmetry; Bilinear model; Electricity spot prices; Log-likelihood ratio test.

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1 Introduction

ts, both in the short run for the purpose of market trading and risk management, and in the longer run for the purpose of investment decision-making and long-term management. In sharp contrast to regulated markets in the past, electricity

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spot prices are now characterized by several stylized facts: infrequenAccurate modeling and forecasting of the spot price distribution is important to market participant and large jumps, excessive leptokurticty when compared to the normal distribution, volatility clustering, and changes in electricity prices tend to be positively correlated with changes in volatility.

The aim of this study is to provide further empirical detail of the stochastic structure of electricity spot price changes in Australia. Specifically, this paper tests the hypothesis that electricity spot price changes adjust asymmetrically to past information. For this purpose, a bilinear model with BL-GARCH errors is introduced and estimated for three Australian electricity markets.

The paper is organized as follows. The next section discusses the data and presents some preliminary findings. Section 3 outlines the model used. Section 4 presents and discusses the main empirical findings, while Section 5 provides concluding remarks.

2 Australian data and preliminary analysis

The data employed in this study consist of half-hourly electricity spot prices for NSW, QLD and VIC, from January 1, 2006, to June 30, 2006, giving 8506 observations for each electricity market. Half-hourly price changes for each market were calculated as the logarithm difference in the half-hourly electricity spot price data, *i.e.*, $R_t = \log P_t - \log P_{t-1}$, where P_t denotes the electricity spot price at time t. The preliminary statistics suggests that the typical features of electricity spot price changes, such as fat tails (kurtosis larger than that of a Normal distribution which is 3), spiked peaks, and persistence of variance (large $LB^2\,(24)$[1] value) are observed in all Australian electricity markets. Likewise, the Jarque-Bera statistics overwhelmingly rejected Normality for all three markets at the 1% level of significance. Further, the calculated LB statistic at the twenty-fourth lags is significant for all electricity spot price changes. Evidence of higher order temporal dependences is provided by the LB statistic when applied to the squared price changes. It can be seen that, for the squared price changes, this statistic is, in general, several times greater than the LB calculated for the price changes, suggesting that higher moment temporal dependences are more pronounced. This, of course, is an empirical regularity encountered in almost all electricity spot price time series, especially in high frequency. What is not clear from these statistics is the extent to which the first and second moment dependences are symmetric.

[1]The Ljung-Box statistic for N lags is calculated as $LB(N) = T(T+2)\sum_{j=1}^{N} \rho_j^2/(T-j)$, where ρ_j is the sample autocorrelation for j lags, and T is the sample size.

3 Models specification and parameter estimation

Let R_t , $t = 1, \cdots, n$ the half-hourly electricity price changes. To model the asymmetry in mean, we consider the $\mathrm{BL}(1,0,1,1)$ given by

$$R_t = a_0 + a_1 R_{t-1} + d_1 R_{t-1}\varepsilon_{t-1} + \varepsilon_t, \tag{1}$$

where, the error process (ε_t) in Equation 1 is assumed to be conditionally heteroskedastic with time varying variance given by

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 + c_1 h_{t-1}\varepsilon_{t-1}, \tag{2}$$

where a_0, a_1, d_1 are constants and c_1 controls the degree of asymmetry. If $c_1 = 0$,ε_t is the error term.

Sufficient conditions for the positivity of the conditional variance h_t^2 have been provided in the paper by Storti and Vitale (2003). For the stationarity condition to hold, we require $\alpha_1 + \beta_1 < 1$.

Given initial values for ε_t and h_t^2, the parameter vector $\omega = (a_0, a_1, d_1, \alpha_0, \alpha_1, \beta_1, c_1, \theta)$ can be estimated by maximizing the log-likelihood over the sample period. The latter can expressed as

$$L(\omega) = \sum_{t=1}^{T} \log f(\mu_t, h_t, \theta), \tag{3}$$

where, μ_t, h_t and θ are the conditional mean, the conditional standard deviation, and other distributional parameters characterizing the conditional density $f(.)$, respectively[2]. In the present paper, the density used is that of the Student-t with degrees of freedom, $\theta = \nu$, estimated endogenously[3]. The advantage of using a more general distribution is that parameter estimates are not excessively influenced by extreme observations which occur with small probability (e.g., electricity price spikes or jumps).

[2]Estimation of the model is done numerically.

[3]The functional form of the Student-t density function is given by

$$f(\mu_t, h_t, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi(\nu-2)}}\left[1 + \frac{\varepsilon_t^2}{(\nu-2)h_t^2}\right]^{-\frac{\nu+1}{2}},$$

where $\Gamma(.)$ is the gamma function, and ν $(\nu > 2)$ a scale parameter, or degrees of freedom to be estimated endogenously. For $\nu > 4$, the conditional kurtosis equals $\frac{3(\nu-2)}{\nu-4}$, which extends the Normal value of three, but for $\nu \to \infty$, the Student-t distribution yields the Normal distribution.

4 Evidence in first and second moment asymmetries

4.1 Analysis of the mean

With the exception of the VIC electricity market, the asymmetric conditional mean response parameter is clearly significant. The hypothesis that $d_1 = 0$ was also tested on the basis of the likelihood ratio test[4]. The estimated LR statistics were well above the critical value at the 10% level, thereby rejecting the hypothesis that the conditional mean is a symmetric function of past returns, with the exception of VIC electricity market.

4.2 Analysis of the variance

As anticipated, the estimated parameters for the conditional variance, α_1, β_1 and c_1, reveal that in all three markets, with no exception, volatility is an asymmetric function of past innovations, suggesting the presence of an "inverse leverage effect". More specifically, good news or positive shocks to prices increase volatility more than bad news or negative shocks.

The estimated degrees of freedom, ν, are very close to three in all markets, suggesting that the kurtosis is at the boundary of being infinite, or is indeed possibly infinite. This confirms the earlier statement: that departures from normality observed in the raw price changes series cannot be entirely attributed to temporal first- and second-moment dependences.

Given that the Ljung-Box statistic, LB^2, does not provide any indication as to how well the model captures the impact of positive and negative innovations on volatility, we use a set of diagnostics proposed by Engle and Ng (1993). These tests are based on the new impact curve implied by the particular ARCH-type model used. The premise is, that if the volatility process is correctly specified, then the squared standardized residuals should not be predictable on the basis of observed variables. The results obtained show no evidence of misspecification, thus supporting rejection of the hypothesis of symmetry in the conditional variance for all of these markets.

[4] The likelihood ratio statistic (LR) was calculated as $LR = -2\,(L_R - L_U)$, where L_R is the value of the log likelihood under the null hypothesis, and L_U is the log likelihood under the alternative. This statistic is asymptotically distributed as χ^2 with degrees of freedom equal to the number of restrictions under the null hypothesis.

5 Conclusion

This paper was motivated by demonstrated empirical evidence of asymmetry in both the conditional mean and conditional variance of (transformed) electricity spot prices. We presented evidence from the literature that judiciously use the bilinear model (in both mean and variance equations) can capture stylised facts of electricity prices. Our approach generalised the method of Storti and Vitale (2003).

Our results have provided measures for the proportionate rise of volatility during market spot price increases, the so-called inverse leverage effect. Our models also allow non-linearity to be employed fully in volatility forecasting. These benefits, with flow-on policy determinations over production and pricing, are due to our new discoveries of the separately identifiable non-linear behaviours in the mean and the variance of the price changes. Specifically, we have harnessed the natural and well-documented mechanics of bilinear time series models to capture the associated sudden bursts in mean and in variance in an informative way which admits predictions. Hence, our results can be exploited by companies and traders for modelling, forecasting, and improving bidding strategies; by regulators, for identifying possible anomalous pricing behaviours; and by hedgers, for the pricing of energy derivatives.

References

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