A model for correlated damage indicators in reliability, with application to maintenance

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Since a few decades, the development of on-line systems observation has allowed to think of enhanced models in reliability, based on the effective measurement of a system deterioration level. In the common case of an increasing deterioration measured at successive times, classical models include coumpound Poisson or Gamma processes, according to whether deterioration is due to isolated shocks or accumulating continuous wear, see [1], [4] or [5] e.g.. Both of these models are univariate non-decreasing Lévy processes. For such models, one single indicator is supposed to synthetize all the system deterioration. This clearly is a simplifying assumption, whereas modern on-line observation often allows to measure several indicators at the same time. These indicators may stand for different aspects of one single system (vertical and lateral alignment defects of a railway track e.g. [3]) or may correspond to different systems located at the same place and subject to common stresses. In each case, these indicators are correlated and a multivariate model is required, which takes into account their dependance.

In this talk, we shall restrict ourselves to the case of multivariate non-decreasing deterioration, which has not been paid that much attention in the literature yet. Based on the fact that they have been previously seen to be well adapted in the univariate case, we propose to use non-decreasing Lévy processes in the multivariate case too. Such processes are also called subordinators. We shall first exhibit different properties of these multivariate subordinators, which make them well adapted for modelling accumulating deterioration. In our applicative context, random variables of interest are reaching times of a specific zone (the failure zone), which stand for times to failure. As an example of "good" property for reliability, we shall observe that reaching times present some aging property, which is some expected property for a time to failure. We shall also point out some restrictions implied by a Lévy process, such as restrictive dependence between margins. We shall finally illustrate the practical use of multivariate subordinators in reliability through the study of a simple classical maintenance policy: a block replacement policy. The influence of the dependence between the marginal wear indicators will be clearly pointed out for the optimal block replacement policy, as well as the influence of the shape of the failure zone. As in [2] where another maintenance policy is studied, this demonstrates the usefulness of taking into account the dependence between the different deterioration indicators, but also shows some intricate correlation between the shape of the failure zone, the dependence between the wear indicators and the optimal policy.

Keywords: Reliability; Multivariate deterioration; Multivariate subordinators; Lévy processes; Preventive maintenance policy.

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